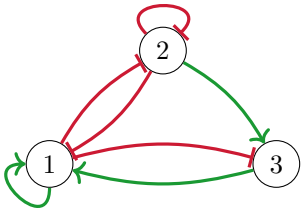
Boolean Networks in Life Sciences
Exercise Sheet 6: Model Verification

Friday 12th December, 2025

Exercise 1 *Consider the following Boolean network of dimension* 3*.*



$$f_1(\mathbf{x}) = \neg\mathbf{x}_2 \vee (\mathbf{x}_1 \wedge \mathbf{x}_3)$$
$$f_2(\mathbf{x}) = \neg\mathbf{x}_1 \vee \neg\mathbf{x}_2$$
$$f_3(\mathbf{x}) = \neg\mathbf{x}_1 \wedge \mathbf{x}_2$$

Characterise the set $\mathcal{S}(000)$ *using the maximal traces.*

Exercise 2 *Sketch the execution tree rooted in* 000 *for the Boolean network from Exercise 1.*

Exercise 3 *Determine the validity of the following LTL formulae in all configurations of the Boolean network from Exercise 1:*

1. $\mathbf{x}_1 \,\mathbf{U}\, \mathbf{x}_3$*;*
2. $\mathbf{G}(\mathbf{F}(\mathbf{x}_2))$*;*

Exercise 4 *Determine the validity of the following CTL formulae in all configurations of the Boolean network from Exercise 1:*

1. $\exists\mathbf{x}_1 \,\mathbf{U}\, \mathbf{x}_3$*;*
2. $\exists(\exists\mathbf{F}(\mathbf{x}_3)) \,\mathbf{U}\, (\forall\mathbf{G}(\mathbf{x}_2))$*;*

Exercise 5 *Assume a Boolean network of dimension at least* 3*. Write LTL formulae that best capture the following properties:*

1. *Eventually, either* 1 *activates or* 2 *stays active forever;*

2. *It remains possible to activate* 3 *at least until simultaneous activation of both* 1 *and* 2*;*

3. 1 *cannot activate until any activation of* 2 *causes immediate deactivation of* 3*;*

Exercise 6 *Assume a Boolean network of dimension at least* 2*. Write CTL formulae that best capture the following properties:*

1. *There exists a successor from which* 1 *is always active until it becomes possible to immediately activate* 2*;*

2. *It is always possible to eventually ensure that* 1 *will always be active infinitely often;*

3. *Exactly one of* 1 *and* 2 *is active across all successors, and there is always a possibility for the active one to stay active forever;*