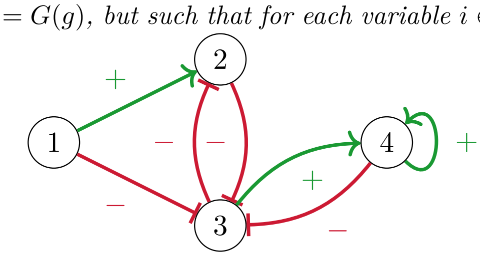
Boolean Networks in Life Sciences
Exercise Sheet 4: Interaction Graphs

Friday 21st November, 2025

Exercise 1 *Construct the interaction graph for each of the following Boolean networks.*

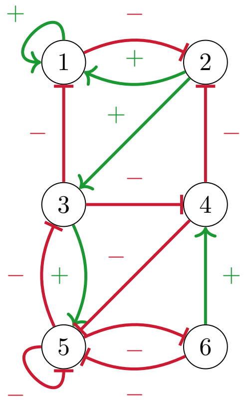
1. $f_1(\mathbf{x}) = \neg \mathbf{x}_2,\ f_2(\mathbf{x}) = \neg \mathbf{x}_1,\ f_3(\mathbf{x}) = \neg \mathbf{x}_1 \wedge \mathbf{x}_2$;
2. $f_1(\mathbf{x}) = \mathbf{x}_3,\ f_2(\mathbf{x}) = (\mathbf{x}_1 \wedge \neg \mathbf{x}_4) \vee (\mathbf{x}_2 \wedge \neg \mathbf{x}_4) \vee (\neg \mathbf{x}_1 \wedge \neg \mathbf{x}_3),\ f_3(\mathbf{x}) = \mathbf{x}_4 \vee (\neg \mathbf{x}_3 \wedge \neg \mathbf{x}_4),\ f_4(\mathbf{x}) = \mathbf{x}_2 \wedge \mathbf{x}_3$;
3. $f_1(\mathbf{x}) = \mathbf{x}_3,\ f_2(\mathbf{x}) = \mathbf{x}_4 \wedge ((\neg \mathbf{x}_1 \wedge \neg \mathbf{x}_3) \vee (\neg \mathbf{x}_1 \wedge \neg \mathbf{x}_5) \vee (\neg \mathbf{x}_3 \wedge \neg \mathbf{x}_5)),\ f_3(\mathbf{x}) = (\neg \mathbf{x}_1 \wedge \mathbf{x}_4) \vee (\neg \mathbf{x}_1 \wedge \mathbf{x}_5) \vee (\mathbf{x}_3 \wedge \mathbf{x}_4 \wedge \mathbf{x}_5),\ f_4(\mathbf{x}) = \mathbf{x}_2,\ f_5(\mathbf{x}) = \neg \mathbf{x}_4 \vee \neg \mathbf{x}_5$;

Exercise 2 *Find two Boolean networks f, g which share the following interaction graph, $G(f) = G(g)$, but such that for each variable $i \in \{1, 2, 3, 4\}$, $f_i \neq g_i$.*

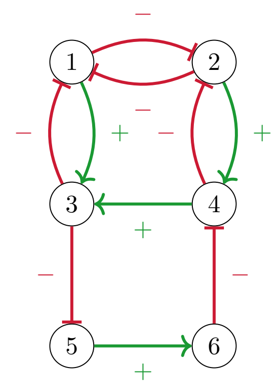


Exercise 3 *For the following interaction graphs, establish the lower and upper bounds for the maximum number of fixed points a Boolean network with the given interaction graph can have.*

1.



2.



Exercise 4 *Find the normal transitions of the following Boolean networks and characterise them by their impact.*

1. $f_1(\mathbf{x}) = \neg\mathbf{x}_1 \vee \neg\mathbf{x}_2,\ f_2(\mathbf{x}) = \neg\mathbf{x}_1 \vee \neg\mathbf{x}_2$;
2. $f_1(\mathbf{x}) = \neg\mathbf{x}_2 \vee \mathbf{x}_3,\ f_2(\mathbf{x}) = \neg\mathbf{x}_1 \vee \mathbf{x}_3,\ f_3(\mathbf{x}) = 1$;
3. $f_1(\mathbf{x}) = \mathbf{x}_1 \vee \mathbf{x}_2,\ f_2(\mathbf{x}) = \mathbf{x}_1 \wedge \mathbf{x}_3,\ f_3(\mathbf{x}) = \neg\mathbf{x}_1 \vee (\mathbf{x}_2 \wedge \mathbf{x}_3)$;
4. $f_1(\mathbf{x}) = \mathbf{x}_3 \vee (\mathbf{x}_1 \wedge \neg\mathbf{x}_2),\ f_2(\mathbf{x}) = \mathbf{x}_3 \vee (\neg\mathbf{x}_1 \wedge \mathbf{x}_2),\ f_3(\mathbf{x}) = \neg\mathbf{x}_3 \wedge (\mathbf{x}_1 \vee \mathbf{x}_2)$;