Boolean Networks in Life Sciences
Exercise Sheet 2: Boolean Networks

Friday 7$^{\text{th}}$ November, 2025

Exercise 1 *Write the definition of each of the eight update functions of a Boolean network of dimension* 3.

Exercise 2 *Consider a Boolean network of dimension* 4. *Define the update functions you would use to implement the following elementary semantics:*

1. *The odd-indexed variables update simultaneously and the even-indexed variables update simultaneously;*

2. *Each variable* $i \in \{1, \ldots, 4\}$ *updates value simultaneously with all variables* $j < i$;

3. *Variable* 2 *and variable* 3 *updates simultaneously with variable* 1. *Variable* 2 *never updates simultaneously with variable* 3. *Variable* 4 *updates simultaneously with exactly one other variable;*

Exercise 3 *Construct the synchronous, fully asynchronous and generalised asynchronous semantics of the following Boolean networks.*

1. $f_1(\mathbf{x}) = \mathbf{x}_2$, $f_2(\mathbf{x}) = \mathbf{x}_1$;

2. $f_1(\mathbf{x}) = \neg\mathbf{x}_2$, $f_2(\mathbf{x}) = \neg\mathbf{x}_1$, $f_3(\mathbf{x}) = \neg\mathbf{x}_1 \wedge \mathbf{x}_2$;

3. $f_1(\mathbf{x}) = \neg\mathbf{x}_2 \vee \mathbf{x}_3$, $f_2(\mathbf{x}) = (\mathbf{x}_2 \wedge \mathbf{x}_1) \vee (\neg\mathbf{x}_2 \wedge \mathbf{x}_3)$, $f_3(\mathbf{x}) = \neg\mathbf{x}_1$;

Exercise 4 *Consider the following fully asynchronous semantics relation and reconstruct the local functions of the Boolean network.*

$000 \to 010$	$100 \to 000$	$010 \to 110$	$101 \to 001$	$011 \to 111$	$111 \to 011$
$000 \to 001$	$100 \to 101$	$110 \to 111$	$101 \to 100$	$011 \to 010$	$111 \to 101$
		$001 \to 011$			