

Boolean Networks in Life Sciences

Exercise Sheet 1: Boolean Algebra

Friday 31st October, 2025

Exercise 1 Construct the truth table for the following Boolean functions.

1. $a \Rightarrow b$;
2. $a \Leftrightarrow b$;
3. $a \oplus b$;
4. $(a \vee \neg b) \Rightarrow (b \Leftrightarrow c)$;
5. $\neg(a \wedge \neg((a \wedge b) \vee \neg c))$

Exercise 2 Find all 16 binary Boolean functions (Boolean functions on two variables).

Exercise 3 Determine whether the following equations hold in Boolean algebra. If so, provide a proof. If not, provide a counterexample.

1. $a \Leftrightarrow b \stackrel{?}{=} (a \Rightarrow b) \wedge (b \Rightarrow a)$;
2. $(a \wedge b) \vee c \stackrel{?}{=} a \wedge (b \vee c)$;
3. $(a \wedge b) \vee (a \wedge \neg b) \stackrel{?}{=} a$;

Exercise 4 Convert the following formulas into both conjunctive normal form and disjunctive normal form.

1. $\neg a \wedge (b \oplus c)$;
2. $(a \Leftrightarrow (b \vee c)) \vee (b \oplus c)$;
3. $\neg(b \vee \neg c) \vee (\neg a \wedge \neg(\neg b \vee c))$;